Strong coupling and long-range collective interactions in optomechanical arrays

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We study cavity optomechanics with an array of equidistant scatterers in a Fabry–Pérot resonator. Our treatment departs from the traditional reflective optomechanics regime with single scatterers and introduces a novel transmissive regime that allows better coupling of light to the collective motional dynamics of the array. We show that the array can behave as a single, highly transmissive, 'superscatterer' possessing a specific collective motional mode which is strongly coupled to the cavity field with an optomechanical coupling strength that scales as $N^{3/2}$ with the number of scatterers N and does not saturate with the reflectivity of the scatterers, as opposed to the reflective case. For the case of small polarisability, *e.g.*, many atoms in an optical lattice, our treatment also uncovers a new regime where the same $N^{3/2}$ scaling is expected.

I. INTRODUCTION AND MOTIVATION

Optomechanics investigates the interaction of light with mechanical oscillators [1, 2] ranging from microscopic ensembles of atoms [3] to macroscopic resonators such as nanomembranes, micromirrors, etc. [1, 2, 4–7] Besides having implications in fundamental quantum mechanics [2], exploiting motion at the quantum level has important applications in metrology [1] and quantum information processing [8]. However, despite recent spectacular progress [1, 2, 9], single-photon coupling is typically very weak for massive resonators, necessitating the use of many photons to amplify the interaction [6]. We introduce the idea that the collective motion of an array of scatterers in a Fabry–Pérot resonator can couple to the cavity field with an interaction strength that scales superlinearly with the number of scatterers and does not saturate with their reflectivity. This new regime should realistically allow for achieving strong single-photon optomechanical coupling [2, 6, 10] with massive resonators and realising quantum optomechanical interfaces [7, 8, 11]. It also sheds light into the long-ranged cooperative optomechanical coupling of atoms or membranes in optical cavities [3, 12, 13].

Cooling a mechanical oscillator to its ground state [9] is but one of a series of achievements that demonstrate the power of coupling light to moving scatterers. (In this article we use terms relevant to the optical domain, but it is to be understood that our conclusions hold equivalently for, *e.g.*, electromechanical systems.) Among the various approaches followed to couple mechanical oscillators with optical resonators, a successful one involves positioning reflecting objects—atoms [3] or dielectric membranes [4, 5]—inside an optical cavity. In the former systems the coupling constant scales as $N^{1/2}$ with the number, N, of atoms [3]. In the latter, it saturates to a fundamental limit g as the reflectivity of the membrane approaches unity [4, 5]. For a highly reflective membrane placed close to the centre of a Fabry–Pérot resonator of length L and resonance frequency ω_0, we can calculate the single-photon coupling strength as the shift in cavity frequency when the mirror moves through a distance equal to the spread, x_0, of its zero-point fluctuations: $g = 2\omega_0 x_0/L$. Several approaches are being followed to reach the strong optomechanical coupling regime in membrane-based systems [4, 5, 14].

Meanwhile, interactions between distant elements in arrays of massive scatterers are believed to be strongly suppressed [13], whereas atomic systems are said to demonstrate infinitely long-ranged interactions [3]. These two systems are two limiting cases of a more general model we describe below. We identify a remarkable 'transmissive' configuration for the scatterers, regardless of whether these are atoms or micromirrors, where the coupling strength (i) scales as $N^{3/2}$ and (ii) does not saturate as the reflectivity of the elements approaches unity. This novel regime allows multi-element opto- or electromechanical systems to reach single-photon coupling strengths orders of magnitude larger than is currently possible. Concomitantly, we show that in this configuration (iii) the resonator field couples to a 'sinusoidal' mechanical mode supporting inter-element interactions that are as long-ranged as the array itself.

Let us again consider a lossless membrane, of thickness much smaller than a wavelength, placed inside a Fabry–Pérot resonator. This time, we suppose that the membrane has an amplitude reflectivity r, which we parametrise in terms of the polarisability $\zeta \equiv -|r|/\sqrt{1-|r|^2}$. The single-photon optomechanical coupling strength is now $g_0 = g|r|$, which is maximised to g for large $|\zeta|$, *i.e.*, in the *reflective* regime $|r| \to 1$. In order to illustrate the emergence of collective optomechanics, we consider now two identical membranes placed symmetrically in the resonator at a distance d from each

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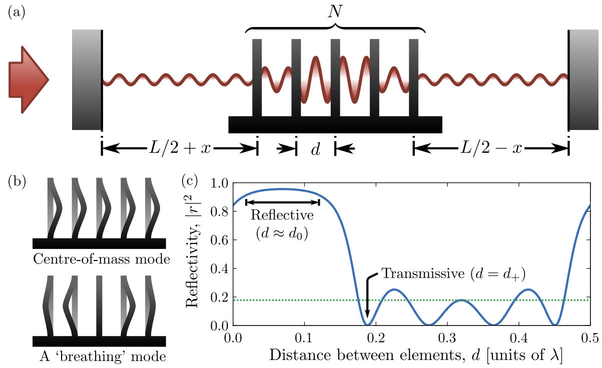

FIG. 1. (Color online) Schematic of our model, motional modes, and working points. (a), System considered: N equidistant scattering elements positioned in the field of a Fabry–Pérot resonator. (b), Two examples of collective motional modes: the centre-of-mass mode, and a representative of a class of 'breathing' modes. (c), Free-space reflectivity of a 5-element array as a function of the inter-element spacing d (in units of λ). The reflectivity of each individual element is 17.6% ($\zeta = -0.5$), illustrated by the dotted green line.

other, in the spirit of Fig. 1(a). As we justify below, the effective polarisability of the two-element system is found to be of the form $\chi = 2\zeta[\cos(kd) - \zeta\sin(kd)]$ for light having wavenumber k. This effective polarisability, and thereby the reflectivity, vanishes when d is chosen such that $kd = \tan^{-1}(1/\zeta) \mod \pi$. Assuming this *transmissive* condition, one can linearise the cavity resonance condition for a small variation δd of the mirror spacing. This readily gives an optomechanical coupling strength

$$g_0' = \left|\frac{\delta\omega}{\delta d}\right| x_0' \approx \sqrt{2}\, g \frac{|r|}{1-|r|}\,, \tag{1}$$

where $x_0' = x_0/\sqrt{2}$ is the extent of the zero-point motion for this breathing mode. It is evident that g_0' scales more favourably with $|r|$ than g_0. One can interpret this result by noting that as the reflectivity of the individual elements is increased, the constructive interference that is responsible for making the array transmissive also strongly enhances the dispersive response of the cavity around this working point. In a symmetric situation the displacement of a mirror in one direction will cause the field to adjust so that the other mirror moves in the opposite direction, thereby balancing the power impinging on the two mirrors. In this simple two-element case, the radiation-pressure force thus couples naturally to a breathing mode [Fig. 1(b)].

Optical 'superscatterers'.—To treat the general case of an array of N equally-spaced elements in free space we make use of the transfer matrix formalism [15] for one-dimensional systems of polarisable scatterers, and derive the response of the system to a propagating light field. As is well-known from the theory of dielectric mirrors, the reflectivity of the ensemble can be tuned to have markedly different behaviors at a given frequency [Fig. 1(c)]. An array of N equally-spaced identical elements, each of polarisability ζ, can be described through the transfer matrix

$$M_N = \begin{bmatrix} (1+i\chi)e^{i\mu} & i\chi \\ -i\chi & (1-i\chi)e^{-i\mu} \end{bmatrix},$$

which relates left- and right-propagating fields on either side of the array. For real ζ it can be shown that N *lossless scatterers behave as a collective 'superscatterer'* having effective polarisability (see Appendix)

$$\chi \equiv \zeta\, \sin[N\cos^{-1}(a)]/\sqrt{1-a^2}\,, \tag{2}$$

with $a = \cos(kd) - \zeta\sin(kd)$, together with a phase shift μ. The ensemble is attains its largest reflectivity for $kd = kd_0 \equiv -\tan^{-1}(\zeta)$, $\chi = \chi_0 \equiv -i\sin\left[N\cos^{-1}\left(\sqrt{1+\zeta^2}\right)\right]$, and becomes fully transmissive ($\chi = 0$) for $kd = kd_\pm \equiv -\tan^{-1}(\zeta) \pm \cos^{-1}\left[(1+\zeta^2)^{-1/2}\cos(\pi/N)\right]$. These are the two working points illustrated in Fig. 1(c).

The phase shift μ, which is the phase accrued on reflection from the stack, can be expressed in terms of the Chebyshev polynomials of the second kind, $U_n(x)$ (see Appendix):

$$e^{i\mu} = \frac{1 - i\zeta U_{N-1}(a)}{(1-i\zeta)U_{N-1}(a) - e^{ikd}U_{N-2}(a)}\,.$$

From this point onwards, the array can be treated as a single scatterer, keeping in mind the dependence of χ on the inter-element spacing.

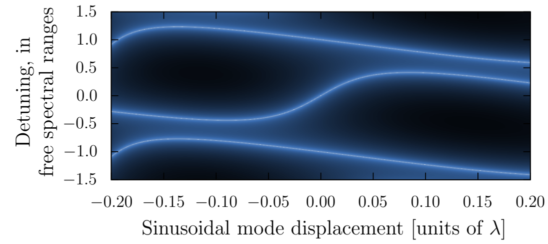

FIG. 2. (Color online) Transmission through a cavity with 5 elements. The sinusoidal mode, a type of breathing mode [Fig. 1(b)], is defined in the text. Note the heightened sensitivity of the resonance at the centre of the figure. ($\zeta = -0.5$, $L = 6.7\,\text{cm}$, $d = d_+$, bare cavity finesse $\approx 3\times10^4$.)

II. OPTOMECHANICAL COUPLING STRENGTH OF THE ENSEMBLE

When placed inside a cavity, cf. Fig. 1(a), this array of scatterers modifies the resonance condition, such that the resonances of the system are given by the solutions to (see Appendix)

$$e^{ikL} = \frac{e^{-i\mu}}{1+i\chi}\left[i\chi\cos(2kx) \pm \sqrt{1+\chi^2\sin^2(2kx)}\right]. \tag{3}$$

These resonances show up when plotting transmission spectra of the system, *e.g.*, as the bright curves in Fig. 2

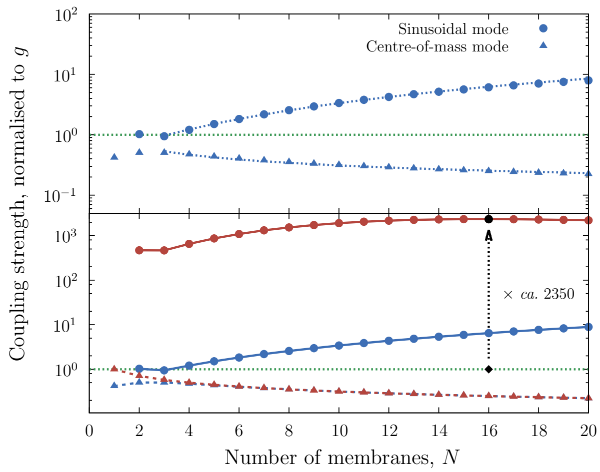

FIG. 3. (Color online) Coupling strengths for multi-element arrays. All these curves are scaled to $g \approx 2\pi \times 15\,\mathrm{Hz}$ (dotted green line). Top: Scaling, with the number of elements N, of the normalised coupling strength for the sinusoidal ($\propto N^{3/2}$) and centre-of-mass ($\propto N^{-1/2}$) modes, as illustrated by the dotted curves. ($\zeta = -0.5$, $L = 6.7\,\mathrm{cm}$, $d = d_+ + 20\,\lambda$.) Bottom: Comparing coupling strengths for the sinusoidal (solid lines) and centre-of-mass (dashed) modes, with $\zeta = -0.5$ (17.6% reflectivity; blue) and $\zeta = -12.9$ (99.4%; red). ($d = d_+$.)

in the case of the sinusoidal mode. Generically, one obtains the linear optomechanical coupling strength by linearising this equation about one of its solutions. For a centre-of-mass motion [cf. Fig. 1(b)] in the reflective regime, $d = d_0$, we thus obtain $g_{\mathrm{com}} = g\sqrt{\mathcal{R}/N}$, where $\mathcal{R} = \chi_0^2/\left(1+\chi_0^2\right)$ is the maximal power reflectivity of the ensemble. As N or ζ increase, $\mathcal{R}$ saturates to 1 and g_{com} scales as $N^{-1/2}$. This scaling can be explained simply by noting that the motional mass m_N of N elements is N times that of a single one; the single-photon coupling strength, which is proportional to $1/\sqrt{m_N}$, therefore *decreases* with N.

In the transmissive regime, $d = d_+$, the intensity profile peaks at the centre of the array [Fig. 1(a)]. The optomechanical coupling strength g_j for the j^{th} membrane is strongest where the difference in amplitudes across the membrane is greatest, $j \approx (N+2)/4$ or $(3N+2)/4$, resulting in $g_j \propto \sin[\pi(2j-1)/N]$. The cavity field therefore couples to the sinusoidal mode, whose profile varies sinusoidally across the array, with a collective coupling strength ($N > 2$) (see Appendix)

$$g_{\sin} = \sqrt{\frac{N}{2}}\,\frac{g\,|\zeta|\csc\!\left(\frac{\pi}{N}\right)\left[\sqrt{\sin^2\!\left(\frac{\pi}{N}\right)+\zeta^2}+|\zeta|\right]}{1+2N\frac{d}{L}|\zeta|\csc^2\!\left(\frac{\pi}{N}\right)\sqrt{\sin^2\!\left(\frac{\pi}{N}\right)+\zeta^2}}\,.$$

or, for large N,

$$g_{\sin} = \frac{\frac{\sqrt{2}}{\pi}\,g\,\zeta^2 N^{3/2}}{1+\frac{2}{\pi^2}\frac{d}{L}\zeta^2 N^3} \approx \frac{\sqrt{2}}{\pi}\,g\,\zeta^2 N^{3/2}\,, \qquad (4)$$

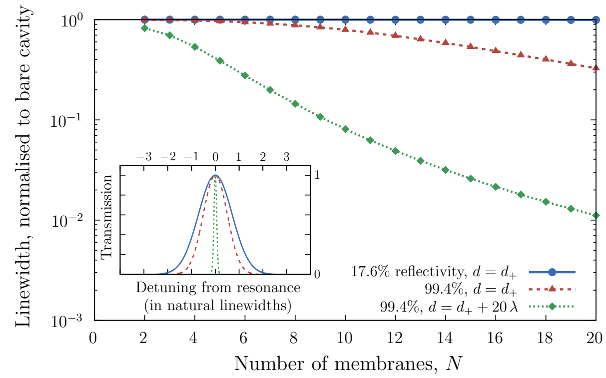

FIG. 4. (Color online) Linewidth-narrowing effect. The effect becomes significant as the reflectivity of the individual membranes increases, and is more pronounced for widely-spaced membranes. Inset: Representative transmission profiles for $N = 10$ for the parameters in the main figure. ($L = 6.7\,\mathrm{cm}$.)

the last expression is valid for $L/d \gg 2\zeta^2 N^3/\pi^2$. Optimising over N for arbitrary L/d, we obtain $g_{\sin}^{\mathrm{opt}} = \frac{1}{2}g\sqrt{L/d}|\zeta|$. This favourable scaling with both N and $|r|$, as shown in Fig. 3, is a significant improvement over the state of the art. Equation (4) is valid both for arrays of membranes ($|\zeta| \gtrsim 1$) and of atoms ($|\zeta| \ll 1$). For typical experiments with cold atoms in cavities [3], $N|\zeta| \ll 1$, *i.e.*, the atoms do not significantly modify the mode structure of the cavity resonance. In this case, $g_{\sin}$ reduces to the usual $N^{1/2}$ scaling that arises from the independent coupling of well-localised atoms interacting with an unmodified cavity field [3]. However, our approach allows us to treat the case of a confined, optically dense [16], and well-localised atomic ensemble having $N|\zeta| \gtrsim 1$ [17], which would exhibit a superlinear $N^{3/2}$ scaling, signalling the breakdown of independent coupling.

An interesting effect arises in the regime where $g_{\sin}$ saturates and eventually starts decreasing as a function of N; the scatterers then act to narrow the cavity resonance substantially. This has possible applications in hybrid systems, along the same lines as those of electromagnetically-induced transparency in Ref. [11], and arises from an effective lengthening of the cavity, due to the presence of the array, to a length $L_{\mathrm{eff}} \equiv L + \frac{2}{\pi^2}d\zeta^2 N^3$. From the fact that the cavity finesse in the transmissive regime is fixed by the end mirrors, it follows that the linewidth of the cavity is $\kappa_{\mathrm{eff}} \propto 1/L_{\mathrm{eff}}$, cf. Fig. 4. When using low-finesse cavities and low mechanical oscillation frequencies, this effect could thus be used to place the system well within the sideband-resolved regime.

III. NUMERICAL EXAMPLE

The power of this approach to optomechanics is best seen through a numerical illustration. If we take commercial silicon nitride membranes [4] with a power reflectiv-

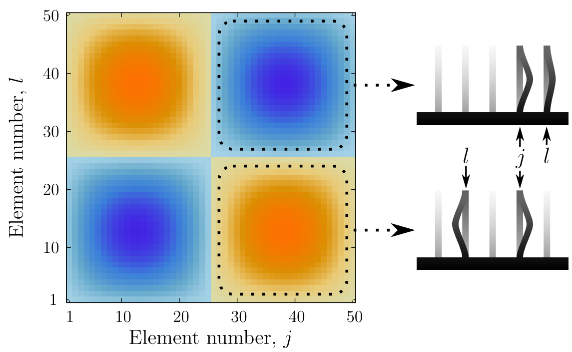

FIG. 5. (Color online) Matrix of inter-element forces. The long range of the interaction between elements of the array can be illustrated by calculating the force experienced by any element (horizontal axis) due to the motion of any other element (vertical axis); for a symmetric system such as ours this matrix is also symmetric. The strongest interactions correspond to the darkest blue (in-phase) or orange (out-of-phase) regions.

ity of 17.6% ($\zeta = -0.5$) and $x_0 = 1.8\,\mathrm{fm}$, and a cavity with $L = 6.7\,\mathrm{cm}$ and ω_0 equivalent to a wavelength of 1064 nm, we can estimate $g_{\mathrm{com}} \approx 2\pi \times (12.8 \times N^{-1/2}\,\mathrm{Hz})$ for $N \gtrsim 3$. For the sinusoidal mode, and with the same parameters, $g_{\mathrm{sin}} \approx 2\pi \times (1.3 \times N^{3/2}\,\mathrm{Hz})$ for large N; an improvement by over an order of magnitude when $N = 10$. A transparent ensemble potentially provides a much stronger optomechanical coupling than a reflective one; indeed $g_{\mathrm{sin}}/g_{\mathrm{com}} \propto N^2$. Let us now consider highly-reflective membranes [14] having 99.4% reflectivity ($\zeta = -12.9$), $x_0 = 2.7\,\mathrm{fm}$, and $\omega_{\mathrm{m}} = 2\pi \times 211\,\mathrm{kHz}$. For $L = 0.25\,\mathrm{cm}$ and $d = d_+$, one obtains $g_{\mathrm{com}} \approx 2\pi \times 600\,\mathrm{Hz}$ and $g_{\mathrm{sin}} \approx 2\pi \times 270\,\mathrm{kHz} > \omega_{\mathrm{m}}$ ($N = 5$); strong coupling between a single photon and a single phonon is already within reach with only very few elements.

Long-range collective interactions.—The collective nature of the interaction that is responsible for these large coupling strengths also gives rise to an effective 'non-local' interaction between the scatterers, where the motion of any particular element influences greatly elements further away, and not just its nearest neighbours. The cavity field mediates an effective interaction between pairs of elements (indices l and j, say) through a coupling constant proportional to $g_l g_j$, illustrated as a 'force matrix' in Fig. 5. This coupling is macroscopically long-ranged, and our system therefore provides an ideal means to gain insight into collective optomechanics phenomena [13]. By contrast, in the reflective regime the light does not permeate through the ensemble, and the force matrix would instead be almost diagonal.

IV. LIMITATIONS

Our model makes three key assumptions about the physical make-up of the membrane ensemble, which we will now discuss in brief. (i) The membranes are assumed to be thin on the scale of a wavelength. Since silicon nitride membranes have thicknesses of the order of $50\,\mathrm{nm} \ll \lambda \sim 1\,\mu\mathrm{m}$ [4], this assumption is not expected to be a limiting factor.

(ii) Positioning errors are also a concern. Piezoelectric actuators can be used to position the membranes with sub-nm accuracy. Nevertheless, let us estimate the positioning requirements in a fairly demanding setup ($N = 10$, $\zeta = -0.5$). The reflectivity of the ensemble reaches 10% when the spacing between each pair of membranes is off by $\sim 10^{-3}\,\lambda \approx 1\,\mathrm{nm}$. This estimate is conservative, since it assumes that the spacing is uniformly incorrect throughout the entire ensemble. If the error in the position of any element is independent of that of any other such that the effects add up 'incoherently,' the expected tolerance required is not beyond current technology.

(iii) The membranes were also assumed to be non-absorbing; this is an excellent approximation to a single membrane with an imaginary part of the refractive index being $\lesssim 10^{-6}$–10^{-5} [5, 18]. In the transmissive case, the optical field permeates throughout the entire ensemble; it is precisely this fact that gives rise to the collective dynamics we described above. The power absorbed by the ensemble therefore increases ($\propto N^{\approx 2}$) with N, and even moderate absorption can seem prohibitive for large ensembles of membranes. This is mitigated by the large coupling strengths obtained, which allow much smaller photon numbers to be used: $g_{\mathrm{sin}}^2 \propto N^3$ increases faster than the power absorbed as N increases. We note further that at large input powers it might be possible to exploit the photothermal force to further enhance the collective optomechanical interaction [14, 19, 20].

V. CONCLUSIONS

We have shown that by operating in the transmissive regime of an ensemble of polarisable scatterers, one couples to a collective mechanical mode strongly enough to bring single-photon optomechanics [10] and strong-coupling effects [6] within reach. The system also gives rise to inter-element interactions that span the entire array. Our ideas apply generically across a wide range of systems; any system that can be modelled as a one-dimensional chain of scatterers (*e.g.*, membranes, atoms [16], optomechanical crystals [9], or dielectric microspheres [21]) is amenable to a similar analysis and shows the same rich physics. Similar methods would allow the extension of these ideas to more complicated systems where the polarisability is a function of frequency or of position along the array, or systems involving the interaction of arrays of refractive elements with multiple optical modes.

VI. ACKNOWLEDGEMENTS

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Appendix A: Transfer matrix for N-membrane stack

Let us consider N equally-spaced identical non-absorbing membranes, each of which has a thickness much smaller than an optical wavelength. The spacing d between these membranes determines the overall optical properties of the ensemble. We shall treat the system as a strictly one-dimensional system, and we shall use the transfer matrix formalism [15, 22]. Our starting point is the matrix that links the fields interacting with a single membrane of polarisability ζ ($\zeta \in \mathbb{R}$ for a lossless membrane),

$$M_{\mathrm{m}}(\zeta) \equiv \begin{bmatrix} 1+i\zeta & i\zeta \\ -i\zeta & 1-i\zeta \end{bmatrix}, \tag{A1}$$

and the matrix that describes propagation of a monochromatic beam of wavenumber k over a distance d through free space,

$$M_{\mathrm{p}}(d) \equiv \begin{bmatrix} e^{ikd} & 0 \\ 0 & e^{-ikd} \end{bmatrix}, \tag{A2}$$

both of which have unit determinant. We wish to evaluate a product of the form

$$M_{\mathrm{m}}(\zeta) \cdot M_{\mathrm{p}}(d) \cdot M_{\mathrm{m}}(\zeta) \cdots M_{\mathrm{m}}(\zeta), \tag{A3}$$

with N copies of $M_{\mathrm{m}}(\zeta)$. First, we note that

$$\begin{aligned} &M_{\mathrm{p}}(d/2) \cdot M_{\mathrm{m}}(\zeta) \cdot M_{\mathrm{p}}(d) \cdots M_{\mathrm{m}}(\zeta) \cdot M_{\mathrm{p}}(d/2) \\ &\quad = \left[M_{\mathrm{p}}(d/2) \cdot M_{\mathrm{m}}(\zeta) \cdot M_{\mathrm{p}}(d/2)\right]^N \equiv M^N, \end{aligned} \tag{A4}$$

where the second line defines the matrix M:

$$M \equiv \begin{bmatrix} (1+i\zeta)e^{ikd} & i\zeta \\ -i\zeta & (1-i\zeta)e^{-ikd} \end{bmatrix}. \tag{A5}$$

We can easily see that $\det M = 1$, whereby it can be shown [23] that for real ζ we can write

$$M^N = \begin{bmatrix} (1+i\chi)e^{i(kd+\mu)} & i\chi \\ -i\chi & (1-i\chi)e^{-i(kd+\mu)} \end{bmatrix}, \tag{A6}$$

where $\chi \equiv \zeta U_{N-1}(a)$, with $U_n(x)$ being the n^{th} Chebyshev polynomial of the second kind, $a = \cos(kd) - \zeta \sin(kd)$, and

$$e^{i\mu} = \frac{1 - i\zeta U_{N-1}(a)}{(1-i\zeta)U_{N-1}(a) - e^{ikd}U_{N-2}(a)}. \tag{A7}$$

Upon removing the padding of $d/2$ from either side, we obtain the matrix that describes the N-membrane ensemble:

$$M_{\mathrm{p}}[\mu/(2k)] \cdot M_{\mathrm{m}}(\chi) \cdot M_{\mathrm{p}}[\mu/(2k)]. \tag{A8}$$

Thus, as stated in the main text, N lossless membranes behave as a collective 'supermembrane' of polarisability χ along with a phase shift $\mu/2$ on either side of the stack.

Appendix B: Resonances of the compound system

Following the main text, let us now place our membrane stack inside a perfect Fabry–Pérot cavity. Finding the resonances for the compound optical system is seemingly trivial; we need only solve the relation

$$\begin{aligned} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \propto \begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix} \times \begin{bmatrix} 1+i\chi & i\chi \\ -i\chi & 1-i\chi \end{bmatrix} \\ \times \begin{bmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{bmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \end{aligned} \tag{B1}$$

with $\theta \equiv k(L/2+x) + \mu/2$ and $\phi \equiv k(L/2-x) + \mu/2$. Here x is the displacement of the ensemble with respect to the centre of the cavity. We thus obtain

$$e^{ikL} = \frac{e^{-i\mu}}{1+i\chi}\left[i\chi\cos(2kx) \pm \sqrt{1+\chi^2\sin^2(2kx)}\right] \tag{B2}$$

However, we immediately see that this equation is transcendental in k, and therefore cannot be solved analytically; this equation is easily solvable for L, and we therefore know the resonant length of our cavity. It is now a legitimate question to ask: 'If d (or x) shifts by a small amount, how much will the resonant frequency of *this* cavity shift?' This question is, of course, easily answered by expanding Eq. (B2) in small increments about its solution. Assuming a dominantly linear effect, we replace $k \to k_0 + \delta k$, $x \to x + \delta x$, $\chi \to \chi + \delta\chi$, and $\mu \to \mu + \delta\mu$ in Eq. (B2). Around resonance, the result simplifies to

$$\begin{aligned} L\delta k + \delta\mu = &\left[-1 \pm \cos(2k_0x)\Big/\sqrt{1+\chi^2\sin^2(2k_0x)}\right] \\ &\times \delta\chi/\big(1+\chi^2\big) \\ &\mp \left[2\chi\sin(2k_0x)\Big/\sqrt{1+\chi^2\sin^2(2k_0x)}\right] \\ &\times (x\delta k + k_0\delta x). \end{aligned} \tag{B3}$$

Appendix C: Centre-of-mass coupling strength

Let us start from Eq. (B3). For the centre-of-mass motion, $\partial\mu = \partial\chi = 0$, and we assume that $|L/x|$ is very large, such that we can write

$$L\delta k = \mp\left[2\chi\sin(2k_0x)\Big/\sqrt{1+\chi^2\sin^2(2k_0x)}\right]k_0\delta x\,. \tag{C1}$$

The right-hand-side of this equation is maximised when $\sin(2k_0x) = \mp 1$, whereby

$$L\delta k = 2k_0\big(-\chi/\sqrt{1+\chi^2}\big)\delta x\,. \tag{C2}$$

This is, in absolute value, a monotonically-increasing function of $|\chi|$ and is therefore maximised when χ attains its largest value, $\zeta\,U_{N-1}\big(\sqrt{1+\zeta^2}\big)$, which leads to $g_{\rm com}$ as defined in the main text.

Appendix D: Coupling to each individual membrane

The matrix M_N representing the ensemble can be written, for $1 \le j \le N$,

$$\begin{aligned}&\begin{bmatrix} e^{i\mu_1/2} & 0 \\ 0 & e^{-i\mu_1/2}\end{bmatrix}\begin{bmatrix} 1+i\chi_1 & i\chi_1 \\ -i\chi_1 & 1-i\chi_1\end{bmatrix}\\ &\quad\times\begin{bmatrix} e^{i(\mu_1/2+\nu+kx_j)} & 0 \\ 0 & e^{-i(\mu_1/2+\nu+kx_j)}\end{bmatrix}\\ &\times\begin{bmatrix} 1+i\zeta & i\zeta \\ -i\zeta & 1-i\zeta\end{bmatrix}\begin{bmatrix} e^{i(\mu_2/2+\nu-kx_j)} & 0 \\ 0 & e^{-i(\mu_2/2+\nu-kx_j)}\end{bmatrix}\\ &\quad\times\begin{bmatrix} 1+i\chi_2 & i\chi_2 \\ -i\chi_2 & 1-i\chi_2\end{bmatrix}\begin{bmatrix} e^{i\mu_2/2} & 0 \\ 0 & e^{-i\mu_2/2}\end{bmatrix}\,,\end{aligned} \tag{D1}$$

where μ_1 and χ_1 describe the ensemble formed by the $n_1 = j-1$ membranes to the 'left' of the $j^{\rm th}$, and μ_2 and χ_2 the one formed by the $n_2 = N-j$ membranes to its 'right'. The displacement of the $j^{\rm th}$ element is denoted x_j; all other membranes are in their equilibrium position. In the transmissive regime, to lowest order in kx_j in each entry, the matrix product above can be written, with the above choice for ν,

$$\begin{bmatrix} e^{i\mu}+\alpha x_j & \beta x_j \\ \beta^* x_j & e^{-i\mu}+\alpha^* x_j\end{bmatrix}\,, \tag{D2}$$

where α and β are increments of first order in the relevant displacement [note that the (off-)diagonal terms are complex conjugates of each other; this is different to the case where absorption is nonzero]. When this matrix is substituted into the equation for the resonance condition, the terms involving $\mathrm{Re}\{e^{-i\mu}\alpha\}$ and $\mathrm{Re}\{\beta\}$ drop out entirely *for a symmetric system*, such that it suffices to consider only the imaginary part of the increment. Let us reiterate that this happens only because the off-diagonal terms are complex conjugates of each other; were absorption to be nonzero, this would no longer be the case. Eq. (B3) now simplifies to

$$\frac{\partial k}{\partial x_j} = -\frac{\mathrm{Im}\{\beta + e^{-i\mu}\alpha\}}{L+2d\frac{\partial\chi}{\partial\nu}}\,, \tag{D3}$$

with $\nu = kd$,

$$\begin{aligned}\alpha &= 2ik\zeta\left[e^{i\mu_1}(1+i\chi_1)\chi_2 - e^{i\mu_2}\chi_1(1+i\chi_2)\right]\\ &= 2ik\zeta^2\Bigg[\frac{(1+\zeta^2)U^2_{n_1-1}(a)U_{n_2-1}(a)}{(1-i\zeta)U_{n_1-1}(a) - e^{i\nu}U_{n_1-2}(a)}\\ &\qquad - \frac{(1+\zeta^2)U^2_{n_2-1}(a)U_{n_1-1}(a)}{(1-i\zeta)U_{n_2-1}(a) - e^{i\nu}U_{n_2-2}(a)}\Bigg]\,,\end{aligned} \tag{D4}$$

and

$$\begin{aligned}\beta &= 2k\zeta\left[\chi_1\chi_2 - (1+i\chi_1)(1-i\chi_2)e^{i(\mu_1-\mu_2)}\right]\\ &= 2k\zeta\Bigg\{\zeta^2 U_{n_1-1}(a)U_{n_2-1}(a)\\ &\qquad - \left[1+\zeta^2U^2_{n_1-1}(a)\right]\\ &\qquad\times\frac{(1-i\zeta)U_{n_2-1}(a) - e^{i\nu}U_{n_2-2}(a)}{(1-i\zeta)U_{n_1-1}(a) - e^{i\nu}U_{n_1-2}(a)}\Bigg\}\,.\end{aligned} \tag{D5}$$

These two expressions simplify considerably to yield

$$\begin{aligned}\mathrm{Im}\{\beta + e^{-i\mu}\alpha\} &= 2k\zeta\csc\left(\frac{\pi}{N}\right)\\ &\times\left[\sqrt{\sin^2\left(\frac{\pi}{N}\right)+\zeta^2} - \zeta\right]\sin\left(2\pi\frac{j-\frac{1}{2}}{N}\right)\,,\end{aligned} \tag{D6}$$

and therefore

$$\begin{aligned}g_j &= -2\omega_{\rm c}x_0\frac{\zeta\csc\left(\frac{\pi}{N}\right)\left[\sqrt{\sin^2\left(\frac{\pi}{N}\right)+\zeta^2}-\zeta\right]}{L-2Nd\zeta\csc^2\left(\frac{\pi}{N}\right)\sqrt{\sin^2\left(\frac{\pi}{N}\right)+\zeta^2}}\\ &\qquad\times\sin\left(2\pi\frac{j-\frac{1}{2}}{N}\right)\,.\end{aligned} \tag{D7}$$

As discussed in the main text, the coupling of the collective motion of the membranes to the cavity field is governed by the constant $\sqrt{\sum_{j=1}^N g_j^2}$, such that for $N>2$

$$g_{\sin} = -\sqrt{\frac{N}{2}}\,\frac{g\,\zeta\csc\left(\frac{\pi}{N}\right)\left[\sqrt{\sin^2\left(\frac{\pi}{N}\right)+\zeta^2}-\zeta\right]}{1-2N\frac{d}{L}\zeta\csc^2\left(\frac{\pi}{N}\right)\sqrt{\sin^2\left(\frac{\pi}{N}\right)+\zeta^2}} \tag{D8}$$

because of the relation

$$\sqrt{\sum_{j=1}^N\sin^2\left(2\pi\frac{j-\frac{1}{2}}{N}\right)} = \begin{cases}\sqrt{2} & \text{for } N=2\\ \sqrt{\frac{N}{2}} & \text{for } N>2\end{cases}\,; \tag{D9}$$

this is equal to $g_{\sin}$ as defined in the main text in the appropriate limits.

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